TAMPINES JUNIOR COLLEGE
Physics Tutorial

Measurement (H1 & H2)

Physical Quantities and Units

1. Which list of SI units contains only base units? (J96/P1/1)

   A) Kelvin, metre, mole, ampere, kilogram
   B) kilogram, metre, second, ohm, mole
   C) kilogram, Newton, metre, ampere, ohm
   D) Newton, Kelvin, second, volt, mole (**A**)

Example 1
Express density in terms of SI base units.

Density = Mass / Volume

Unit of density = Unit of (Mass / Volume)
= kg m^{-3}

- What is the definition of density and hence its formula?
- What are the units for the quantities in the formula?

2. For the following quantities, express their units in terms of SI BASE units:
   a) Velocity — m s^{-1}
   b) Acceleration — m s^{-2}
   c) Force — ($F = m\,a$), kg m s^{-2}
   d) Pressure — ($P = F / A$), kg m^{-1} s^{-2}
   e) Work Done — (Work = F x d), kg m^2 s^{-2}
   f) Kinetic Energy — (K.E = ½ $m\,v^2$), kg m^2 s^{-2}
   g) Potential Difference — ($V = W / Q$), kg m^2 A^{-1} s^{-3}
   (Hint: Potential difference between two points is the amount of electrical energy converted to other forms of energy per unit charge flow between the two points.)
   h) Electrical Resistance — ($R = V / I$), kg m^2 A^{-2} s^{-3}

3. Suppose the base quantities chosen had been length (m), force (N) and second (s), what would be the units of the following?
   a) mass — ($F = m\,a$), N m^{-1} s^2
   b) density — (density = m / V), N m^{-4} s^2

4. When a beam of light is incident on a surface, it delivers energy to the surface. The intensity of the beam is defined as the energy delivered per unit area per unit time.
   What is the unit of intensity, expressed in SI base units? (H1 2007/P1/2)

   A) kg m^{-2} s^{-1}
   B) kg m^2 s^{-3}
   C) kg s^{-2}
   D) kg s^{-3} (**D**)

Finding units of unknown quantities

Example 2
In the ideal gas law, $PV = nRT$, what are the possible units of R?
(P = gas pressure,
V = gas volume,
n = number of moles of gas,
T = thermodynamic temperature,
R = molar gas constant)

R = PV / nT (Rearrange equation to make R the subject)

Unit of R = unit of (PV / nT)
= (kg m^2 s^{-2}) / (mol K)
= kg m^2 s^{-2} mol^{-1} K^{-1}
or J mol^{-1} K^{-1}

5. The force F acting on a particle of charge Q moving at velocity v perpendicular to a magnetic field of strength B is given by $F = BQv$. Express B in base units. (Hint: current = charge per unit time)

$B = F / Qv$

Base unit of B = base unit of (F / Qv)
= (kg m s^{-2}) / [(A s) (m s^{-1})]
= kg s^{-2} A^{-1}

6. Coulomb's law states that $F = Q_1Q_2 / 4\pi\varepsilon_o r^2$, where Q_1, Q_2, are charges experiencing an electrostatic force F between them, and r is the distance between their centres. Express the unit of ε_o in terms of the base units.

Make ε_o the subject of the equation. [A^2 s^4 m^{-3} kg^{-1}]

7. A radio aerial of length L, when the current is I, emits a signal of wavelength λ and power P. These quantities are related by

$$P = kI^2\left(\frac{L}{\lambda}\right)^2$$

where k is a constant.
What unit, if any, should be used for the constant k? (H2 2007/P1/2)

A) volt
B) ohm
C) watt
D) no unit (**B**)

Ratio of L and λ has no units. $P = I^2R$

8. When a body moves through a fluid, a retarding force due to turbulence may be experienced. In the case of a sphere of radius r moving with speed v through a particular fluid of density ρ which is at rest, this force is given by

$$F = k \rho r^2 v^2$$

Show that k is unitless.

(Make k the subject and then observe that the units of quantities on the RHS will cancel out.)

Example 3

The experimental measurement of the heat capacity C of a solid as a function of temperature T is to be fitted to the expression $C = \alpha T + \beta T^3$. What are the possible units of α and β? (C has units J K^{-1})

Given LHS: the unit of C = J K^{-1} and assuming the equation $C = \alpha T + \beta T^3$ is homogenous, then

RHS:

unit of αT = J K^{-1} Therefore, unit of α = J K^{-2}

unit of βT^3 = J K^{-1} Therefore, unit of β = J K^{-4}

*9. In a certain experiment to measure the viscosity η of a liquid of density ρ, the time t taken for the liquid level to fall a given distance in the apparatus is measured. The viscosity η is given by:

$$\eta = A\rho t - B\rho / t$$

where A and B are constants to be determined in a calibration experiment.

a) The usual unit for η is N s m^{-2}. Show that this unit may be expressed as kg m^{-1} s^{-1}.

b) If η is in N s m^{-2}, find the units of A and B in the above equation.

Answer: unit of A = m^2 s^{-2}, unit of B = m^2

**10. The mass M of the largest stone which can be moved by the water of a flowing river is assumed to be

$$M = k v^p \rho^q g^r \text{ where}$$

k = unitless constant,
v = velocity of water flow,
ρ = density of water flow,
g = acceleration due to gravity,
and p, q and r are unitless constants.

Determine p, q and r.

unit of M = units of $k v^p \rho^q g^r$ (homogenous equation)

kg = $(m\ s^{-1})^p (kg\ m^{-3})^q (m\ s^{-2})^r$

= m^{p-3q+r} s^{-p-2r} kg^q

Comparing units and powers of LHS and RHS,

q = 1

p – 3q + r = 0
p – 3 + r = 0

-p – 2r = 0

Solving simultaneously to get p = 6, r = -3, q = 1

Example 4
A theory suggests that *R*, the resistance of a sample of a certain intrinsic semiconductor depends on *T*, the thermodynamic temperature by the equation

$$R = A\, e^{(B/T)}$$

where A and B are constants. Find the units of A and B.

Since $e^{(B/T)}$ is unitless,
unit of A = unit of *R*
= Ω

B/*T* is unitless, therefore
unit of B = unit of *T*
= K

11. The equation relating current *I* through a semiconductor diode to the applied potential difference *V* at temperature *T* is

$$I = I_o \exp\left(-\frac{qV}{kT}\right)$$

where *q* is the electron charge, and k is a constant.

If I_o is a characteristic constant of the diode with the same units as current *I*, find the units of k.

- Index has no units
- Definition of p.d, *V* = *W*/*Q*
[J K^{-1} or kg m^2 s^{-2} K^{-1}]

Estimates of Physical Quantities

12. Give an SI unit and an estimate of the magnitude of each of the following physical quantities.

	Magnitude	**Unit**
The weight of an adult		
The power of a hair dryer		
The energy required to bring the boil a kettleful of water		
The resistance of a domestic filament lamp		

The wavelength of visible light		

(J99/P2/1)

(Marks will be awarded for the correct order of magnitude of each estimate, not for its accuracy.)

Ans: 10^2 N, 10^3 W, 10^5 W, 10^3 Ω, 10^{-7} m

13. What is a reasonable estimate for the volume of a wooden metre rule found in a school laboratory? (H2 2007/P1/1)

A) 1.5 cm^3
B) 15 cm^3
C) 150 cm^3
D) 1500 cm^3 (**C**)

Typical length = 1 m = 100 cm
Typical breadth = 3 cm
Typical height = 0.5 cm
Hence, estimate for volume = … = 150 cm^3

*14. Using the ideal gas equation $pV = NkT$ where p = gas pressure, V = gas volume, N = number of gas molecules, k = Boltzmann constant = 1.38 x 10^{-23} J K^{-1} and T = thermodynamic temperature, estimate the number of gas molecules in the classroom on a typical afternoon.
(Hint: you should be able to make rough estimates of pressure, temperature and volume of air in the classroom)

Assume the air in the room behaves ideally,
Atmospheric pressure in classroom, p = 1 x 10^5 Pa
Thermodynamic temperature, T = 273 + 27
= 300 K
Volume of air in classroom =10 x 10 x 3
= 300 m^3

Therefore number of gas molecules, N = pV / kT
= $(10^5)(300) / (1.38 \times 10^{-23})(300)$
= 7 x 10^{27}

Prefixes of Units

15. Complete the table below to show each prefix with its corresponding decimal equivalent.

prefix	**decimal equivalent**
pico	10^{-12}
micro	10^{-6}
giga	10^9
tera	10^{12}

Unit conversion

16. Convert from km h^{-1} to m s^{-1}
a) 600 km h^{-1} =
[167 m s^{-1}]
Convert from g cm^{-3} to kg m^{-3}
b) 3.5 g cm^{-3} =
[3500 kg m^{-3}]

Experimental Errors

17. Errors in measurement may be either systematic or random. Which of the following involves random error? (N2003/P1/2)

A) not allowing for zero error on a moving-coil voltmeter
B) not subtracting background count rate when determining the count rate from a radioactive source
C) stopping a stopwatch at the end of a race
D) using the value of *g* as 10 N kg^{-1} when calculating weight from mass

(**C**)

18. Which of the following experimental techniques reduces the systematic error of the quantity being investigated? (J92/P1/1)

A) Timing a large number of oscillations to find the period
B) Measuring the diameter of a wire repeatedly and calculating the average
C) Adjusting an ammeter to remove its zero error before measuring a current
D) Plotting a series of force and extension readings for a spring which obeys Hooke's Law on a graph and using its gradient to find the spring constant k.

(**C**)

19. A micrometer, reading to ±0.01 mm, gives the following results when used to measure the diameter *d* of a uniform wire:

1.02 mm 1.02 mm 1.01 mm 1.02 mm 1.02 mm

When the wire is removed and the jaws are closed, a reading of –0.02 mm is obtained.
Which of the following gives the value of *d* with a precision appropriate to the micrometer? (J2000/P1/2)

A) 1.0 mm
B) 1.00 mm
C) 1.038 mm
D) 1.04 mm

(**D**)

Accuracy and Precision

Example 5
The following table shows 4 groups of 5 measurements of *g*, the acceleration of free fall, in m s^{-2}. Which group of measurement obtained a set of results that could be described as both precise and accurate?

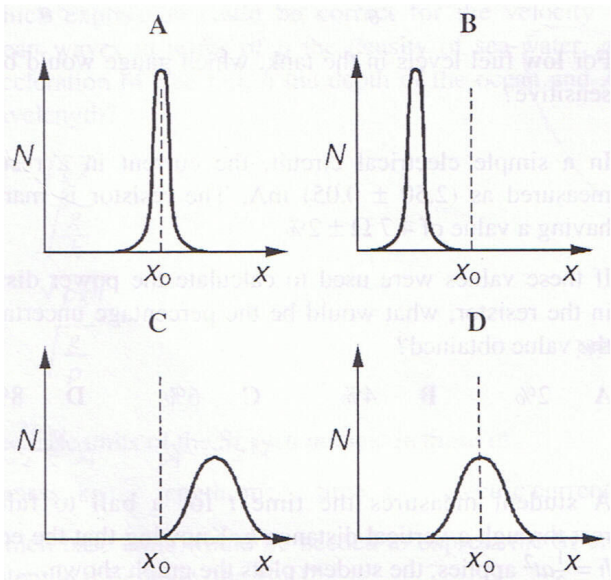
	1	2	3	4	5	Ave	Precise?	Accurate?
Group 1	9.70	9.45	9.20	9.99	10.60	9.79	×	✓
Group 2	9.65	9.65	9.66	9.67	9.67	9.66	✓	×
Group 3	9.81	9.80	9.77	9.80	9.78	9.79	✓	✓
Group 4	9.11	9.44	9.56	9.25	9.03	9.28	×	×

Group 3 is both precise and accurate.

For accuracy, look at which set has an average value closer to the actual value of *g*.
For precision, look at which set has a smaller range (i.e. difference between biggest and smallest values).

20. A quantity *x* is measured many times and the number *N* of measurements giving a value *x* is plotted against *x*. The true value of the quantity is x_o.

Which graph best represents precise measurements with poor accuracy? (N97/P1/2)



(**B**)

21. A voltmeter connected across a resistor in a circuit gives readings which have high precision but low accuracy.

Which of the following best describes the likely error in readings taken with this voltmeter? (H1 2007/P1/1)

	random error	systematic error
A	high	high
B	high	low
C	low	high
D	low	low

(**C**)

Expressing numerical values to required number of significant figures or decimal places for calculations not involving uncertainties

Rules of thumb

- When performing addition and subtraction on quantities, the final result, as a rule of thumb, should be quoted in the least number of decimal places of the given quantities. (See example 6)
- When performing multiplication and division on quantities, the final result, as a rule of thumb, should be quoted in the least number of significant figures of the given quantities. (See example 7)
- Note that the d.p. / s.f. of constants are ignored when determining the precision of the final result. (See example 7)

Example 6
If x = 20.0 cm, y = 6.943 cm and z = 1.1004 cm. Evaluate L where $L = x + y + z$.

L = $x + y + z$
= 20.0 + 6.943 + 1.1004
= 28.0434
= 28.0 cm (1 d.p.)

Example 7
Suppose a = 38.500, b = 1.36, c = 1.6, evaluate

(a) $ab / 5c$ = (38.5000 x 1.36)/ (5 x 1.6)
= 6.545
= 6.5 (2 s.f.)

(b) $6a$ = 6 x 38.500
= 231.00 (5 s.f.)

(c) $4b^5$ = 4 x $(1.36)^5$
= 18.6103
= 18.6 (3 s.f.)

22. Evaluate the product of 5.6 and π.

[18, rounded to 2 s.f.]

23. If a = 3.40, b = 0.020, c = 4.0, evaluate $2a + bc$.

$2a + bc = 2(3.40) + (0.020)(4.0) = 6.80 + 0.080 = 6.88$
(2 s.f) (2 d.p)

Absolute, Fractional and Percentage Uncertainty

24. A student takes the following readings of the diameter of a wire with micrometer screw gauge: 1.52 mm, 1.48 mm, 1.49 mm, 1.51 mm, 1.49 mm, 1.51 mm. How would you express the diameter of the wire together with its uncertainty?

Average diameter = 1.50 mm
Maximum deviation from average value = 0.02 mm
Therefore, diameter = (1.50 ± 0.02) mm
Note: Absolute uncertainty to be written as 1 s.f. and d.p. of final result and absolute uncertainty must match.

25. The length of a piece of paper is measured as 297 ± 1 mm.
    a) What is the fractional uncertainty in its length?
    [0.00337]
    b) What is the percentage uncertainty in its length? (N89/P2/2)
    [0.337%]

26. Suppose $Y = 10\,p$, and p carries an uncertainty of $\pm\Delta p$.
    a) Does Y have the same absolute uncertainty as p? No
    b) Does Y have the same percentage uncertainty as p? Yes
    c) The thickness of a metal sheet was measured after it was twice folded by half. Compared to measuring the thickness of the sheet directly, is the percentage uncertainty in the value of the thickness thus obtained reduced?
    No, it's still the same. Can conclude that in general, if $C = n\,p$, where n is a fraction or integer, $\Delta C = n\,\Delta p$, but $\Delta C/C = \Delta p/p$

27. A student makes measurements from which he calculates speed of sound as 327.66 m s^{-1}. He estimates that his result is accurate only to ± 3%. Which one of the following gives his results reduced to the appropriate number of significant figures? (J84/P2/1)

    A) 300 m s^{-1}
    B) 327 m s^{-1}
    C) 327.7 m s^{-1}
    D) 328 m s^{-1}
    E) 330 m s^{-1} (**E**)

Error $\Delta v = 0.03 \times 327.66 = 9.83 \approx 10$ (1 s.f.)
Hence v = (330 ± 10) m s^{-1} [or $(3.3 \pm 0.1) \times 10^2$ m s^{-1}]

Uncertainty in a derived quantity

For functions involving **addition or subtraction,** the uncertainty in a derived quantity is given by the **sum of all separate absolute uncertainties**.

Example 8

Suppose $x = (3.7 \pm 0.1)$ cm and $y = (1.24 \pm 0.02)$ cm.

a) Given that $A = x + y$, find the uncertainty of A.

$A = x + y$
$= 3.7 + 1.24$
$= 4.94$ cm

$\Delta A = \Delta x + \Delta y$
$= 0.1 + 0.02$
$= 0.12$
$= 0.1$ cm (1 s.f.)

Therefore, $A = (4.9 \pm 0.1)$ cm

b) Given that $B = 10x - 3y + 5$, find the uncertainty of B.

$B = 10x - 3y$
$= 10(3.7) - 3(1.24)$
$= 33.28$ cm

In general, if $C = n\,p$, where n can be a fraction or integer, then $\Delta C = n\,\Delta p$

$\Delta B = 10\Delta x + 3\Delta y$
$= 10(0.1) + 3(0.02)$
$= 1.06$
$= 1$ cm (1 s.f.)

Therefore, $B = (33 \pm 1)$ cm

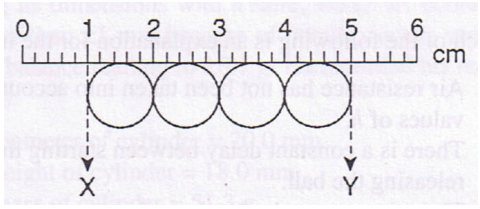
28. The lengths of three sides, a, b, c of a triangle are (2.1 ± 0.1) cm, (2.8 ± 0.1) cm and (3.1 ± 0.1) cm respectively.
    a) The perimeter of the triangle is p where $p = a + b + c$. Express p together with its uncertainty. [(8.0 ± 0.3) cm]
    b) The quantity q is given by $q = 6a + 3b - 2c$. Express q together with its uncertainty. [(15 ± 1) cm]

29. In an experiment, the external diameter d_1 and internal diameter d_2 of a metal tube are found to be (64 ± 2) mm and (47 ± 1) mm respectively. The percentage error in $(d_1 - d_2)$ expected from these readings is at most

    A) 1% B) 5% C) 6% D) 18%

    (**D**)

30. A student attempts to measure the diameter of a steel ball by using a metre rule to measure four similar balls in a row.



The student estimates the positions on the scale to be as follows:

X	(1.0 ± 0.2) cm
Y	(5.0 ± 0.2) cm

What is the diameter of a steel ball together with its associated uncertainty? (J94/P1/2, J99/P1/2)

A) (1.0 ± 0.05) cm
B) (1.0 ± 0.1) cm
C) (1.0 ± 0.2) cm
D) (1.0 ± 0.24) cm (**B**)

For functions involving **multiplication or division,** the **fractional uncertainty** in a derived quantity is given by the **sum of all separate fractional uncertainties**.

Example 9
If $E = p^3 = p \times p \times p$

Then $\Delta E/E = \Delta p/p + \Delta p/p + \Delta p/p = 3\Delta p/p$

In general, if $E = p^n$, where n can be a fraction or integer, then $\Delta E/E = n\ \Delta p/p$

Example 10
If $F = 5p^3q^2 / r^{½}$, obtain an expression for the uncertainty of F.

$\Delta F/F = 3\Delta p/p + 2\Delta q/q + ½\Delta r/r$
$\Delta F = (3\Delta p/p + 2\Delta q/q + ½\Delta r/r) \times F$
$= (3\Delta p/p + 2\Delta q/q + ½\Delta r/r) \times 5p^3q^2 / r^{½}$

Since the exact value 5 does not have an uncertainty, it also does not contribute to the fractional uncertainty of F.

Example 11

Suppose $x = (3.7 \pm 0.1)$ cm and $y = (1.24 \pm 0.02)$ cm.

a) Given that $D = 5x^3 / y^2$, find the fractional uncertainty of D and hence write the value of D with its uncertainty.

$D = 5x^3 / y^2$
$= 5(3.7)^3 / (1.24)^2$
$= 164.714$ cm

Check the equation to derive the units of D!

$\Delta D/D = 3\,\Delta x/x + 2\,\Delta y/y$
$= 3(0.1/3.7) + 2(0.02/1.24)$
$= 0.11$

$\Delta D = 0.11 \times 164.714$
$= 20$ cm (1 s.f.)

D.p. of final value follows that of the uncertainty.

Therefore, $D = (160 \pm 20)$ cm or $(1.6 \pm 0.2) \times 10^2$ cm or (1.6 ± 0.2) m

31. The lengths of three sides, a, b, c of a triangle are (2.1 ± 0.1) cm, (2.8 ± 0.1) cm and (3.1 ± 0.1) cm respectively.

a) The quantity r is given by $r = abc$. Express r with its uncertainty. [(18 ± 2) cm^3]

b) The quantity s is given by $s = 2a^6b^3 / c^2$. Express s together with its uncertainty. [(400 ± 200) cm^7]

32. A student uses vernier calipers to measure the sides of a rectangular wafer of silicon to the nearest tenth of a millimetre. The lengths are 10.4 mm, 6.3 mm and 2.3 mm.

Which of the following best expresses the volume of the wafer? (N92/P1/2)

A) 1.50696×10^2 mm^3
B) 1.5070×10^2 mm^3
C) 1.507×10^2 mm^3
D) 1.51×10^2 mm^3
E) 1.5×10^2 mm^3 (**E**)

$V = LBH = 150.696$ mm^3

$$\frac{\Delta V}{V} = \frac{\Delta L}{L} + \frac{\Delta B}{B} + \frac{\Delta H}{H} = \left(\frac{0.1}{10.4}\right)\left(\frac{0.1}{6.3}\right)\left(\frac{0.1}{2.3}\right) = 0.06897$$

$$\Delta V = \left(\frac{\Delta V}{V}\right)V = (0.06897)(150.696) = 10.39 \approx 10$$ (1 s.f.)

Hence $V = (150 \pm 10)$ mm^3 or 1.5×10^2 mm^3.

*Example 12

The equation governing the volume rate of flow, V/t, of a fluid under streamline conditions through a horizontal pipe of length l and radius r is

$$\frac{V}{t}=\frac{\pi pr^4}{8l\eta}$$

where p is the pressure difference across the pipe and η is the viscosity of the fluid. In an experiment to find η for water, a student quotes his result as 1.137×10^{-3} kg m^{-1} s^{-1} and estimates the percentage uncertainties in his measurements of V/t, p, l and r as ±3%, ±2%, ±0.5%, and ±5% respectively. How should he have written the value of η together with its uncertainty?

$$\frac{V}{t}=\frac{\pi pr^4}{8l\eta}$$

To find the uncertainty of η, make η the subject of the equation.

$$\eta = \frac{\pi pr^4}{8l(V/t)}$$

100% x $\Delta\eta/\eta$ = $\{\Delta p/p + 4(\Delta r/r) + \Delta l/l + [\Delta(V/t) / (V/t)]\}$ x 100%
= 2% + 4(5%) + 0.5% + 3%
= 25.5%

$\Delta\eta$ = η x 25.5 %
= 1.137 x 10^{-3} x (25.5 / 100)
= 0.3 x 10^{-3} kg m^{-1} s^{-1} (1 s.f.)

Hence η = (1.1 ± 0.3) x 10^{-3} kg m^{-1} s^{-1}

33. a) The time t for 20 oscillations of a simple pendulum as recorded on a stopwatch is (27.4 ± 0.1) s. If T is the period (time for one oscillation) of the pendulum, find T and ΔT.

(b) The length l of the pendulum as measured by a ruler is (46.6 ± 0.1) cm. Given

$$T=2\pi\sqrt{\frac{l}{g}}$$

where g is the acceleration of free fall. Express g together with its uncertainty Δg.

[Ans: (1.370 ± 0.005) s, (9.80 ± 0.09) m s^{-2}]

34. The electrical power P dissipated in a resistor is given by $P = I^2R$, where I is the current and R the resistance. If P = (7.5 ± 0.3) W and R = 30 Ω ± 1%, the percentage error in I is at most

A) 0.3% B) 1.5% C) 2.5% D) 3.7%

(**C**)

[make *l* the subject first]

**35. Two students want to measure the speed of sound by the following procedure. One of them, positioned some distance away from the other, sets off a firecracker. The second student starts a stopwatch when he sees the flash and stops it when he hears the bang. The speed of sound is roughly 300 m s^{-1} and the students are aware of an uncertainty of 0.3 s in the time recorded. If they wish to keep the uncertainty in the measured speed of sound to within 5%, what is the distance over which they should perform the experiment? You may assume that the distance can be measured to ± 0.1 m.
Some students may get confused by the last few sentences of the question and pick 'distance' as the subject of the equation instead of 'speed'. Then they will get a negative answer.

Solution:
Speed, s $= d / t$
300 $= d / t$

$\Delta s / s = \Delta d / d + \Delta t / t$
0.05 $= 0.1 / (300\ t) + 0.3 / t$

$t = 6$ s

300 $= d / t$
300 $= d / 6$
$d = 1800$ m

[1800 m]