TAMPINES JUNIOR COLLEGE
JC1 Physics 2008

Measurement (H1 & H2)

Assessment Objectives

Candidates should be able to:

(a) recall the following base quantities and their units: mass(kg), length (m), time (s), current (A), temperature (K), amount of substance (mol).

(b) express derived units as products or quotients of the base units and use the named units listed in 'Summary of Key Quantities, Symbols and Units' as appropriate. (Refer to ***Appendix B***)

(c) show an understanding of and use the conventions for labelling graph axes and table columns. (Will be covered in practical sessions)

(d) use the following prefixes and their symbols to indicate decimal sub-multiples or multiples of both base and derived units: pico (p), nano (n), micro (μ), milli (m), centi (c), deci (d), kilo (k), mega (M), giga (G), tera (T).

(e) make reasonable estimates of physical quantities included within the syllabus.

(f) show an understanding of the distinction between systematic errors (including zero errors) and random errors.

(g) show an understanding of the distinction between precision and accuracy.

(h) assess the uncertainty in a derived quantity by simple addition of actual, fractional or percentage uncertainties (a rigorous statistical treatment is not required).

References

1. Whelen & Hodgson (1989) ESSENTIAL PRINCIPLES OF PHYSICS (2ND ED)
2. Tom Duncan, Physics
3. Resnick & Halliday, Fundamentals of Physics

1 Physical Quantities

A physical quantity is a property that can be measured. The speed of a rocket, the area of a soccer field, and the temperature of a hot cup of coffee are all examples of physical quantities.

To completely express a physical quantity, it must be quoted with 1) *a numerical value* and 2) *a unit.*

E.g. Distance = 50 m

"50" is the numerical value and "m" is the unit. Units are important because the expression "50" is meaningless without it.

All physical quantities in Physics can be classified into:
- (i) base or fundamental quantities and
- (ii) derived quantities.

1.1 Base Quantities and Units

In the SI system of units, there are *seven* base quantities and corresponding units.

Base Quantity		**SI Base Unit**	
Name	**Usual Symbol**	**Name**	**Symbol**
Length,	l	metre	m
Mass	m	kilogram	kg
Time	t	second	s
amount of substance	n	mole	mol
electric current	I	ampere	A
Temperature	T	Kelvin	K
luminous intensity*		candela	cd

*Not required for the H1 or H2 Physics syllabus

These units were chosen based on the principles that they are easily and accurately reproducible and unchanging with time. The definitions of these base units are given in ***Appendix A***.

To make life easier for all, it is agreed that all nations will use the SI (Système International) units as the system of measurement.

1.2 *Derived Quantities and Units*

These are physical quantities which are derived from the seven base quantities by mathematical operations such as multiplication, division.

Their units are similarly derived as products or quotients of the seven base units.

Derived Quantity	From Base Quantities	Derived Unit (SI)	Alternative Unit
Density	mass / $(\text{length})^3$	kg m^{-3}	-
Velocity	length / time	m s^{-1}	-
Momentum = mass x velocity	mass x (length / time)	kg m s^{-1}	-
Force = mass x acceleration	mass x length / $(\text{time})^2$	kg m s^{-2}	Newton (N)
Pressure = Force /area	mass /(length x time^2)	kg m^{-1} s^{-2}	Pascal (Pa)

1.3 *Unitless or Dimensionless Quantities*

A unitless or dimensionless quantity is a ratio of 2 quantities having the same or no units. Common examples include:

1) Relative density of material = density of material / density of water

2) Strain = deformation of material / original length of material

3) refractive index = sin i / sin r

<u>Quick Check 1</u>

Which of the following are possible units of a derived quantity?
i) kg   ii) N   iii) m^2 iv) kg s^{-1}

A) i & ii
B) iii & iv
C) ii, iii & iv
D) All of the above   (C)

1.4 Estimating Physical Quantities

Examples of some reasonable estimates are:

Height of a typical man	1.7 m
Paper thickness	0.1×10^{-3} m
Diameter of a hydrogen atom	10^{-10} m
Diameter of atomic nucleus	10^{-15} m
Mass of a an apple	0.1 kg
Room temperature	300 K
Power of a hair dryer	10^{3} W

Tips on making good estimates:

1. Keep your estimates to one significant figure of precision unless you are confident. Exact values are not expected.
2. Do not estimate derived quantities like area or volume directly. Predict the linear dimensions instead and do your calculations from there.
3. When dealing with complex shapes, idealize them as simpler shapes, e.g. a cube or a sphere.
4. Check your final answer to see if it is reasonable.

1.5 Prefixes of Units

Since physical quantities can take a wide range of values, prefixes such as *kilo*, *centi* and *milli* are used together with units to simplify the expressions for both very large and very small quantities.

Standard Prefixes

Prefix	Multiple	Symbol	Example
pico	10^{-12}	p	10 pF (capacitance)
nano	10^{-9}	n	600 nm (orange light)
micro	10^{-6}	μ	1 μm (fibre optics)
milli	10^{-3}	m	mm (wire diameter)
centi	10^{-2}	c	2 cm (wavelength of microwave)
deci	10^{-1}	d	1 dℓ (1 cup)
kilo	10^{3}	k	60 kg (mass of a man)
mega	10^{6}	M	95.0 MHz (radio freq)
giga	10^{9}	G	500 GB (harddisk capacity)
tera	10^{12}	T	0.15 Tm (distance between Earth and Sun)

Note: except for a few prefixes in the middle, most of the powers differ by a factor of "3" or "-3".

1.6 Physical Equations

Equations which are governed by the laws of Physics must be *homogeneous*, i.e. the units of quantities on both sides of an equation must be consistent. As a result, only terms with the same units can be equated, added or subtracted.

E.g. $v^2 = u^2 + 2as$

> The units of v^2, u^2 and as must be the same for the equation to be homogenous!

Units of v^2 $= (\text{m s}^{-1})^2$ $= \text{m}^2\ \text{s}^2$
Units of u^2 $= (\text{m s}^{-1})^2$ $= \text{m}^2\ \text{s}^2$
Units of as $= (\text{m s}^{-2})(\ \text{m})$ $= \text{m}^2\ \text{s}^2$

Since units of all the quantities on both sides are consistent, the equation is homogenous.

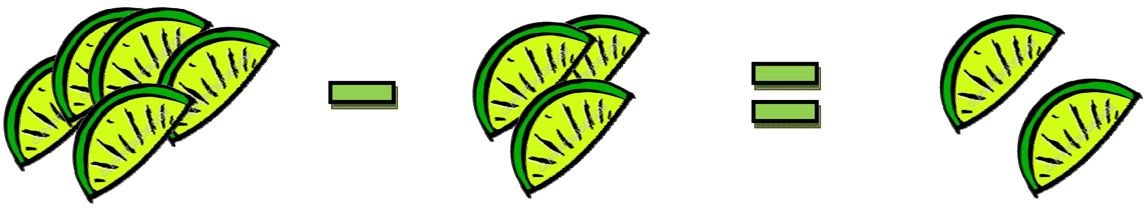
Knowledge of this consistency in equations can be helpful when deriving units of an unknown quantity. You may wish to refer to ***Appendix C*** for further information on the homogeneity of equations.

<u>Quick Check 2</u>

When a steel sphere of radius *r* falls at speed *v* through a liquid, it experiences a drag force *F* given by $F = krv$, where k is a constant. With the aid of the table on the right, which one of the following is a suitable SI unit for k?

Quantity	Units
F	N
r	m
v	m s^{-1}

A) N
B) N s^{-1}
C) $\text{N m}^2\ \text{s}^{-1}$
D) $\text{N m}^{-2}\ \text{s}$ (D)



2 Measurement Techniques

More than one type of instrument may be used to measure the same physical quantity:

Physical Quantity	Instrument
Length	Ruler, vernier calipers, micrometer
Mass	Electronic balance, lever balance
Weight	Spring balance, Newton meter
Time	Stopwatch, calibrated time-base of a Cathode Ray Oscilloscope (CRO)
Temperature	Resistance thermometer, thermocouple
Electric current	Galvanometer, ammeter
Potential difference	Voltmeter, digital multimeter

However, no instrument can claim to measure a physical quantity exactly. There will always be some kind of *experimental error* and a certain degree of *uncertainty*.

2.1 Experimental Errors

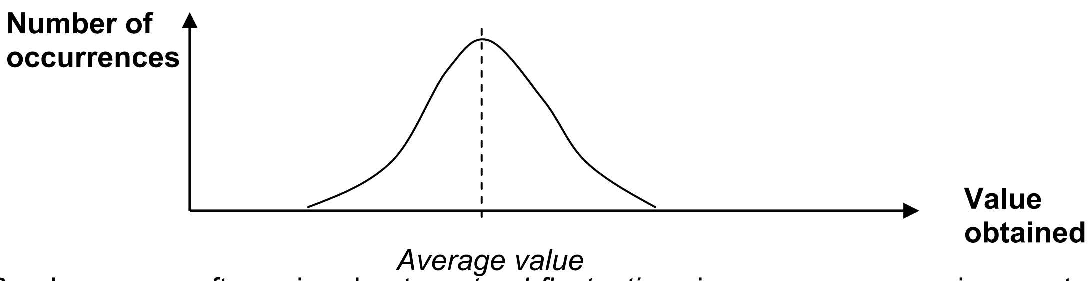
Experimental errors are deviation of measurements from their true values. They are generally divided into two categories, *random* and *systematic*.

Random Error

Random errors are said to occur when repeated measurements of the same quantity give rise to errors of different magnitude and sign.

Random error occurs in an unpredictable manner. It has an equal probability of being positive or negative in sign.

If a measurement is repeated many times and a graph is plotted of the number of occurrences with the values obtained, it may look as follows. There will be a spread of values obtained and the spread occurs around an average value. Such a spread shows the presence of random errors.



Random errors often arise due to *natural fluctuations* in processes or environmental conditions. These include:

- irregularities in specimens (e.g. the diameter of a wire at various points may be slightly different)
- the human reaction time in using stopwatches (e.g. when timing the period of an oscillation, there will be a random error in judging exactly when the oscillation ends)
- the number of decay per unit time measured in a radioactive substance
- changes in temperature or pressure

Although random errors can *never* be eliminated, they can be reduced by:

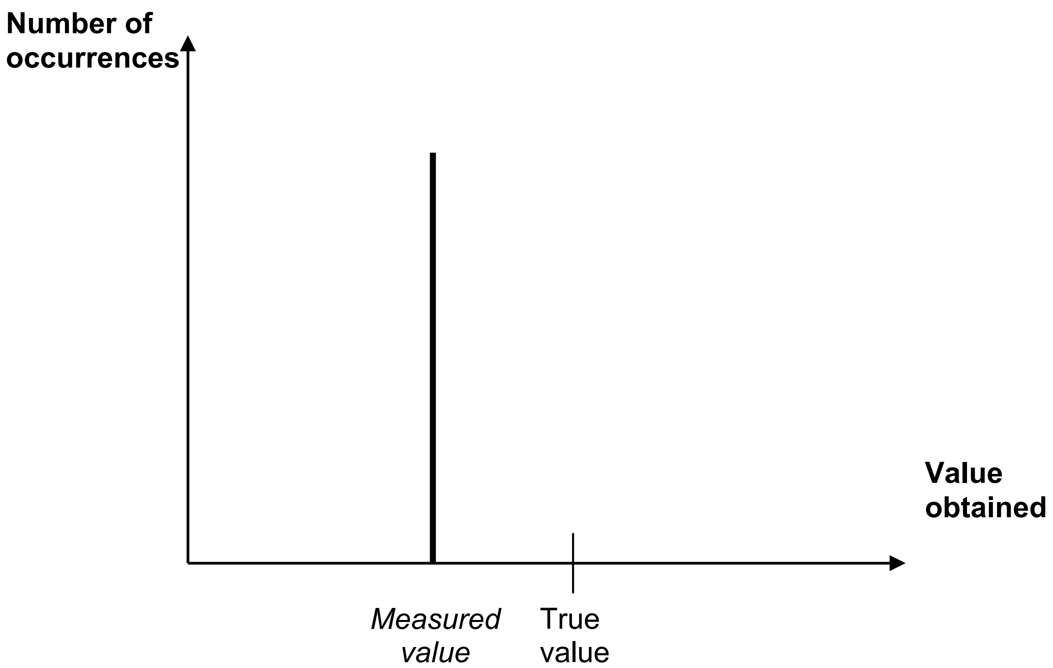
- Taking a large number of readings and then taking the average, or
- Repeating measurements and drawing a line of best fit through the plotted points on a suitable graph.

Systematic Error

Systematic errors are said to occur when repeated measurements of the same quantity under the same conditions give rise to errors of the same magnitude and sign.

Systematic errors are consistent errors that cause all measurements to be incorrect by the same amount. They are predictable since they follow some fixed rule or pattern. Repeating the experiment under the same conditions will yield the same errors.

Graphically, the measurement is shifted from the true value by a fixed amount.



Typical systematic errors include:

- zero errors (e.g. using an ammeter with zero reading of -0.2 A will result in all readings taken to be 0.2 A too small),
- incorrect calibration of instruments,

- bias of the observer,
- incorrect experimental technique (e.g. poor skills resulting in parallax errors that affect all the readings *in the same way*).

Unlike random errors, taking a large number of readings and averaging will *NOT* reduce systematic errors. Fortunately however, systematic errors can be *eliminated* if the cause of that error is known and corrections are made to the readings. For example,

- To correct for zero errors, we subtract the zero reading from all subsequent readings we take.
- For instruments that are suspected to be incorrectly calibrated, we compare them with other instruments that are known to be correctly calibrated.

Summarizing the Differences

	Random Errors	**Systematic Errors**
Magnitude of errors	Variable	Constant
Sign of errors	Equally likely to be positive or negative	Same
Reduced by taking more readings and finding average	Yes	No
Can be totally eliminated	No	Yes

<u>Quick Check 3</u>

Experimental errors can be classified as either random or systematic. How would you classify the human reaction time error involved in an experiment measuring the periodic time of oscillations?

A) It is a random error.
B) It is a systematic error.
C) Require more information about the error (**C**)

2.2 *Accuracy v.s. Precision*

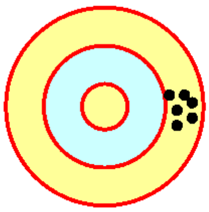
Accuracy and precision are two terms that are often taken to mean the same thing in the English language. However they take on very different meanings in Physics. It is possible to have readings taken with great precision and yet are inaccurate. One likely reason would be the existence of systematic errors. Similarly it is possible to have readings that are accurate but not very precise. This is especially so when the measurements were taken by very basic instruments, resulting in larger than normal random errors.

Simply put,

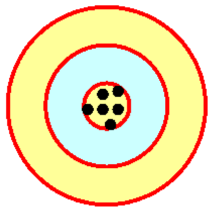
Accuracy refers to how close a reading is to the true value.

Precision refers to how exactly a reading is determined without regard to the true value.

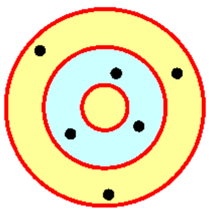
Alternatively, if we consider a target board in which the bull's-eye is the true value, then the way the shots (measured readings) land will indicate the shooter's accuracy and precision. As an illustration:



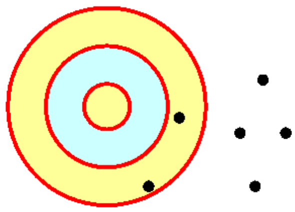
Inaccurate but Precise



Accurate and Precise



Accurate but imprecise



Inaccurate and imprecise

<u>Quick Check 4</u>

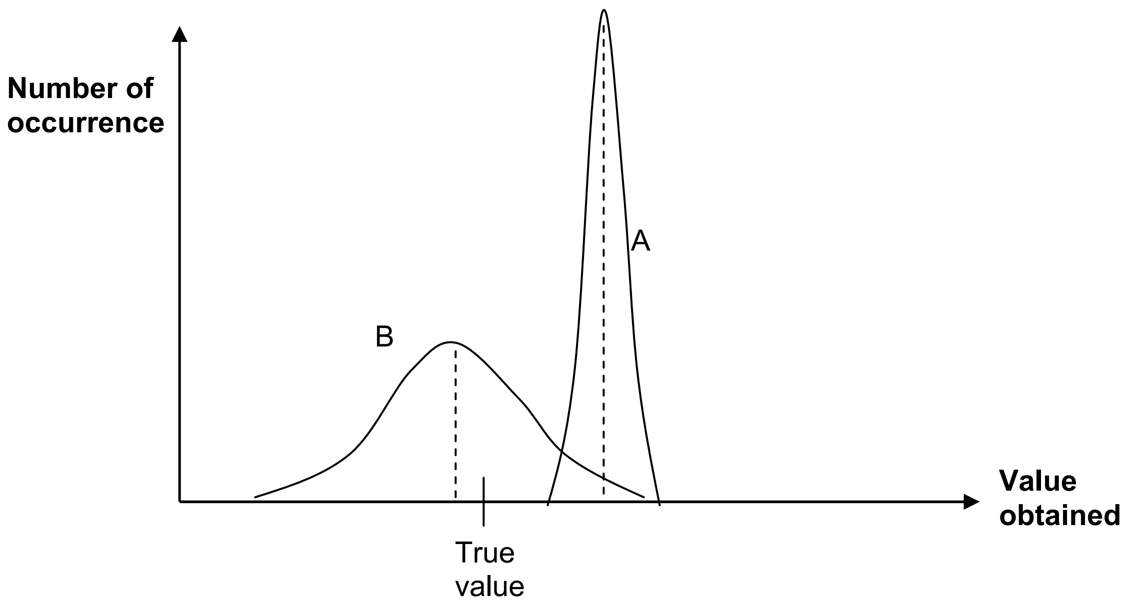
Consider the following two groups of five measurements for the value of g, whose true value is 9.81 m s^{-2}.

	1	2	3	4	5	Ave
Group A	9.70	9.45	9.20	9.99	10.60	9.79
Group B	9.65	9.65	9.66	9.67	9.67	9.66

What group is more accurate? **Group A is more accurate.**

Which group is more precise? **Group B is more precise.**

Quick Check 5



Given the above two sets of experimental results A and B obtained for a particular measured quantity,

which reading is more precise?

Experiment A is more precise than B since it leads to a smaller spread of values than B.

which reading is more accurate?

Experiment B is more accurate than A since the average value obtained is closer to the true value than that for A.

2.3 Significant Figures

The number of digits needed to express the precision of a measurement is called its significant figures. A measurement should *always* be written with only as many digits as are significant. More significant figures generally indicate that a more precise instrument is being used.

Quick Check 6

How many significant figures do the following values of mass have?

Mass/ kg	No. of significant figures
1205	**4**
01205	**4**
0.1205	**4**
12.050	**5**
0.0012050	**5**
12050	**4 or 5**
1.2050×10^4	**5**
1.205×10^4	**4**

In the expression of a value, the rightmost significant figure is the number that carries an uncertainty.

For example when quoting the value 1205 kg, we are telling others that we are unsure of the last digit 5. The true value is equally likely to be slightly more or less than that value, by a number in the *same order of magnitude* as 5.

2.4 Actual (Absolute) Uncertainty

Whenever a measurement is made, the value obtained always carries an uncertainty that is dependent on the precision of the instrument. This uncertainty, called the *absolute uncertainty*, is usually either the smallest division or half the smallest division in the calibration of the instrument. As a result, the instrument used determines the number of decimal places that should be quoted for all measurements made with it.

For example, when a length of exactly 158 mm is measured using a metre rule, the value obtained carries an uncertainty of ± 1 mm. Hence the final length can be quoted as (158 ± 1) mm, or (0.158 ± 0.001) m.

Likewise, if an ammeter enables a reading to be taken to ± 0.02 A, then all readings made from this ammeter will carry 2 decimal places, e.g. (1.20 ± 0.02) A. It should never be written merely as (1.2 ± 0.02) A.

Note:

1. The absolute uncertainty is always given to *1 significant figure only*.
2. The number of decimal places in a value must follow that in the actual uncertainty.
3. Generally, all measurements should be recorded in the form $R \pm \Delta R$ where ΔR is the *absolute uncertainty of R*.

In addition, it is useful to learn the terms:

- *fractional uncertainty,* which is defined as $\Delta R / R$.

and

- *percentage uncertainty,* which is defined as $(\Delta R / R) \times 100\%$.

2.5 Uncertainty of Derived Quantities

The outcome of an experiment is seldom determined by the measurement of a single quantity; More often than not, it is a byproduct of several related quantities. Fortunately, the resulting uncertainty of the final product can be calculated based on the type of operators used to obtain it.

Addition and Subtraction

For ***addition or subtraction,*** the *absolute* uncertainty of a derived quantity is given by the ***sum*** of all the individual *absolute* uncertainties.

Worked Example 1

Given $a = 10 \pm 1$, $b = 25 \pm 3$, find Y if

i) $Y = a + b$,

[Step 1: Find the actual value of Y]

$Y = 10 + 25 = 35$

[Step 2: Find ΔY, making sure only one sig. fig. quoted]

$\Delta Y = \Delta a + \Delta b = 4$

[Step 3: Check same d.p. for both]

Hence, $Y = 35 \pm 4$.

ii) $Y = b - a$,

$Y = 25 - 10 = 15$, $\Delta Y = \Delta a + \Delta b = 4$

Hence, $Y = 15 \pm 4$.

Quick Check 7

If $A = p + q - r$,

Which of the following best represent the uncertainty of A?

A) $\Delta A + \Delta p + \Delta q - \Delta r$

B) $\Delta A - \Delta p + \Delta q + \Delta r$

C) $\Delta p + \Delta q - \Delta r$

D) $\Delta p + \Delta q + \Delta r$ (**D**)

Multiplication and Division

> For ***multiplication or division,*** the *fractional* uncertainty of a derived quantity is given by the ***sum*** of all the individual *fractional* uncertainties.

<u>Worked Example 2</u>

Given $p = 10 \pm 1$, $q = 40 \pm 2$, find D if

i) $D = p\,q$,

[Step 1: Find the actual value of D]

D = 10 x 40 = 400

[Step 2: Find ΔD/D]

Since $\Delta p/p = 0.1$, $\Delta q/q = 0.05$

$\Delta D/D = 0.1 + 0.05 = 0.15$

Caution! The most common mistakes in this type of questions occur due to carelessness in Steps 3 and 4.

[Step 3: Find ΔD, making sure only one sig. fig. quoted]

$\Delta D = 0.15 \times 400 = 60$

[Step 4: Check same d.p. for both]

Hence, D = 400 ± 60.

ii) $D = p/q$,

D = 10/40 = 0.25,

$\Delta D = 0.15 \times 0.25 = 0.0375 \approx 0.04$ (1 sig. fig.)

Hence, D = 0.25 ± 0.04.

<u>Quick Check 8</u>

If $B = 10\,p^2$

Which of the following best represent the uncertainty of B?

A) $\Delta 10 + \Delta p + \Delta p$

B) $\Delta 10/10 + \Delta p/p + \Delta p/p$

C) $\Delta p/p + \Delta p/p$

D) $(\Delta p/p + \Delta p/p)\,B$ (**D**)

In general, if $B = p^n$, $\Delta B/B = n\,\Delta p/p$ (n can be a fraction or integer)

Note also that there is no uncertainty to the exact value of 10.

Appendix A: Definitions of the SI Base Units

1 kilogram		The mass of a piece of platinum-iridium alloy kept under standard conditions near Paris.
2 second		The duration of 9192613770 periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesiurn-133 atom.
3 metre		The distance travelled in 1/299792458 of a second by plane EM waves in a vacuum.
4 Ampere		The electric current which, if maintained in two straight parallel conductors of infinite length and negligible circular cross-section, when placed one metre apart in a vacuum would produce, per metre of length, a force of 2×10^7N between the two conductors.
5 Kelvin		The fraction 1/273.16 of the thermodynamic temperature of the triple point of water.
6 mole		A mole the amount of substance of a system which contains as many molecules, atoms or elementary entities as there are carbon atoms in 0.012kg of carbon-12.

Cover image of "The International System of Units" brochure (8th edition, 2006)

Appendix B: Summary of Key Quantities, Symbols and Units

The following list illustrates the symbols and units that will be used in question papers.

Quantity	***Usual symbols***	***Usual unit***
Base Quantities		
mass	m	kg
length	*l*	m
time	t	s
electric current	I	A
thermodynamic temperature	T	K
amount of substance	n	mol
Other Quantities		
distance	d	m
displacement	s, x	m
area	A	m^2
volume	V, v	m^3
density	ρ	kg m^{-3}
speed	u, v, w, c	m s^{-1}
velocity	u, v, w, c	m s^{-1}
acceleration	a	m s^{-2}
acceleration of free fall	g	m s^{-2}
force	F	N
weight	W	N
momentum	p	N s
work	w, W	J
energy	E,U,W	J
potential energy	E_p	J
kinetic energy	E_k	J
heating	Q	J
change of internal energy	ΔU	J
power	P	W
pressure	p	Pa
torque	T	N m
gravitational constant	G	N kg^{-2} m^2
gravitational field strength	g	N kg^{-1}
gravitational potential	φ	J kg^{-1}
angle	θ	°, rad
angular displacement	θ	°, rad
angular speed	ω	rad s^{-1}
angular velocity	ω	rad s^{-1}
period	T	s
frequency	f	Hz
angular frequency	ω	rad s^{-1}
wavelength	λ	m
speed of electromagnetic waves	c	m s^{-1}
electric charge	Q	C
elementary charge	e	C
electric potential	V	V
electric potential difference	V	V

electromotive force	E	V
resistance	R	Ω
resistivity	ρ	Ω m
electric field strength	E	N C^{-1}, V m^{-1}
permittivity of free space	ε_o	F m^{-1}
magnetic flux	Φ	Wb
magnetic flux density	B	T
permeability of free space	μ_o	H m^{-1}
force constant	k	N m^{-1}
Celsius temperature	θ	°C
specific heat capacity	c	J K^{-1} kg^{-1}
molar gas constant	R	J K^{-1} mol^{-1}
Boltzmann constant	k	J K^{-1}
Avogadro constant	N_A	mol^{-1}
number	N, n, m	
number density (number per unit volume)	n	m^{-3}
Planck constant	h	J s
work function energy	Φ	J
activity of radioactive source	A	Bq
decay constant	λ	s^{-1}
half-life	$t_{1/2}$	s
relative atomic mass	A_r	
relative molecular mass	M_r	
atomic mass	m_a	kg, u
electron mass	m_e	kg, u
neutron mass	m_n	kg, u
proton mass	m_p	kg, u
molar mass	M	kg
proton number	Z	
nucleon number	A	
neutron number	N	

Appendix C: Homogeneity of Equations

- *Homogeneity v.s. Physical Correctness*

A physical equation is *homogeneous* or *dimensionally consistent* if all terms in the equation separated by the plus, minus or equality signs have the same units.

An example: $v = u + at$

A non-example: $v^2 = u + 2as$

All physically correct equations must be homogeneous. However just because an equation is homogeneous does *NOT* mean it will be physically correct (i.e. obeys the laws of Physics). For example,

both $T = 3\pi\sqrt{\frac{L}{g}}$ and $KE = \frac{1}{8}mv^2$ are homogeneous but not physically correct.

- *Checking the Homogeneity of Equations*

This procedure is also known as Dimensional Analysis. The procedure is similar to the Units Analysis shown on page 5. The slight difference is that base symbols instead of base units are used to check on both sides of the equation.

- *Uses and Limitations of Dimensional Analysis*

Uses

1. To sort out incorrect or non-homogeneous equations immediately.
2. To predict or derive possible equations. As mentioned, such equations are only *plausible* and their correctness must be verified either experimentally or mathematically.

Limitations

1. The values of dimensionless constants (without units) like ½, π etc. cannot be found.

   E.g. K.E = k mv^2 k cannot be found because it is dimensionless

2. Combinations of physical quantities whose final product is dimensionless will not be detected.

3. Only relationships involving products or quotient powers can be found. Those involving functions such as sin, cos, tan, log, e^x cannot be found.

Compilation: KCY Edit: IS